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DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS  
SCHOOL OF ENGINEERING  
OLD DOMINION UNIVERSITY  
NORFOLK, VIRGINIA

LARGE-AMPLITUDE MULTIMODE RESPONSE  
OF CLAMPED RECTANGULAR PANELS TO  
ACOUSTIC EXCITATION

By

Chuh Mei, Principal Investigator

Interim Technical Report  
For the period October 1, 1980 - September 30, 1981

Prepared for the  
Air Force Flight Dynamics Laboratory  
Wright-Patterson Air Force Base  
Ohio

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Under  
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Howard F. Wolfe, Program Manager  
AFWAL/FIBED  
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## FOREWORD

This report contains the research effort on large-amplitude multimode response of clamped rectangular panels to acoustic excitation during the period from October 1, 1980 to September 30, 1981. The work was performed at the Department of Mechanical Engineering and Mechanics, Old Dominion University, Norfolk, Virginia. The research was sponsored by the Air Force Office of Scientific Research (AFSC), Department of the Air Force, under Grant AFOSR-80-0107. The work was monitored under the supervision of Howard F. Wolfe, Technical Manager, Acoustics and Sonic Fatigue Group, Air Force Wright Aeronautical Laboratories, and Dr. Alan H. Rosenstein, AFOSR/NE, Program Manager, Directorate of Aerospace Sciences, AFSC. The author gratefully acknowledges the encouragement and assistance from Mr. Howard F. Wolfe and Dr. Donald B. Paul of AFWAL.

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# LARGE AMPLITUDE MULTIMODE RESPONSE OF CLAMPED RECTANGULAR PANELS TO ACOUSTIC EXCITATION

By

Chuh Mei

## INTRODUCTION

Acoustically induced fatigue failures in structural components have resulted in unacceptable maintenance and inspection burdens associated with aircraft and missile operation. In some cases, sonic fatigue failures have resulted in major structural redesigns and aircraft modifications. Thus, accurate prediction methods are needed to determine the fatigue life of structures.

Many analytical and experimental programs to develop sonic fatigue design criteria, however, have repeatedly shown a poor comparison between measured and calculated maximum RMS stress/strain (refs. 1, 2). Deviations in excess of 100 percent are not uncommon. Large deflection nonlinearity has been identified as a major factor for the enormous discrepancy between test data and computed results (ref. 3). A test program was conducted recently to check the analytical effort for the large-amplitude, single-mode response reported in reference 3. The acoustic response tests were performed in the Wideband Acoustic Facility at Wright-Patterson Air Force Base. A comparison of the results from two panels is shown in figure 1. The prediction of random responses is much improved with the single-mode computational method, especially at high excitation levels. Test results (fig. 2) also showed that there are more than one mode responding. Multiple modes were also observed by White in experimental studies on aluminum and carbon fiber-reinforced plastics (CFRP) plates under acoustic loadings (ref. 4). White also showed that the fundamental mode responded significantly and contributed more than 80 percent of the total mean-square strain response; higher modes, up to third or fourth modes, account for 95% or more of the total mean-square strain response. In order to have an accurate prediction of the random response of a structure, multiple modes should be used in the formulation.

# RESULTS COMPARISON

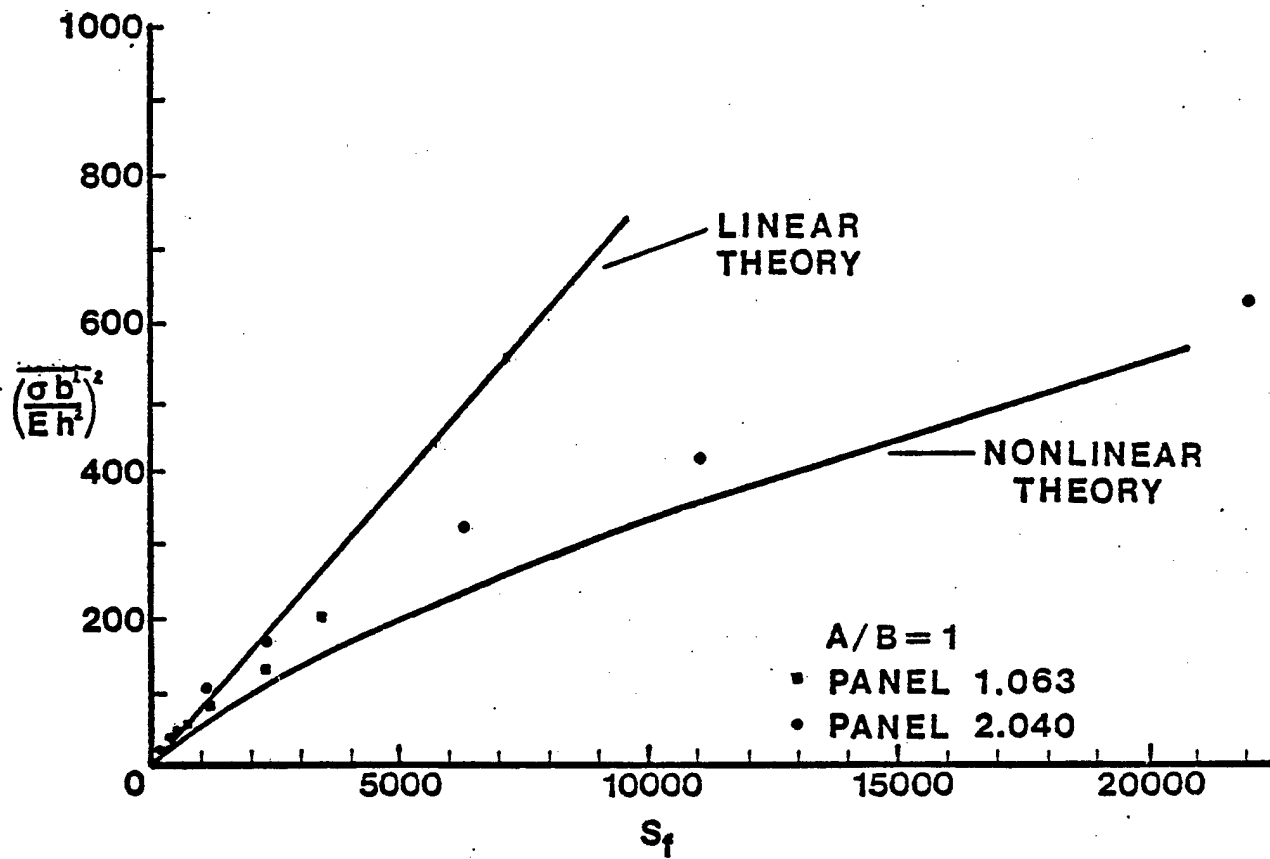


Figure 1. Comparison of analytical and experimental mean-square stresses of clamped, square, aluminum panels.

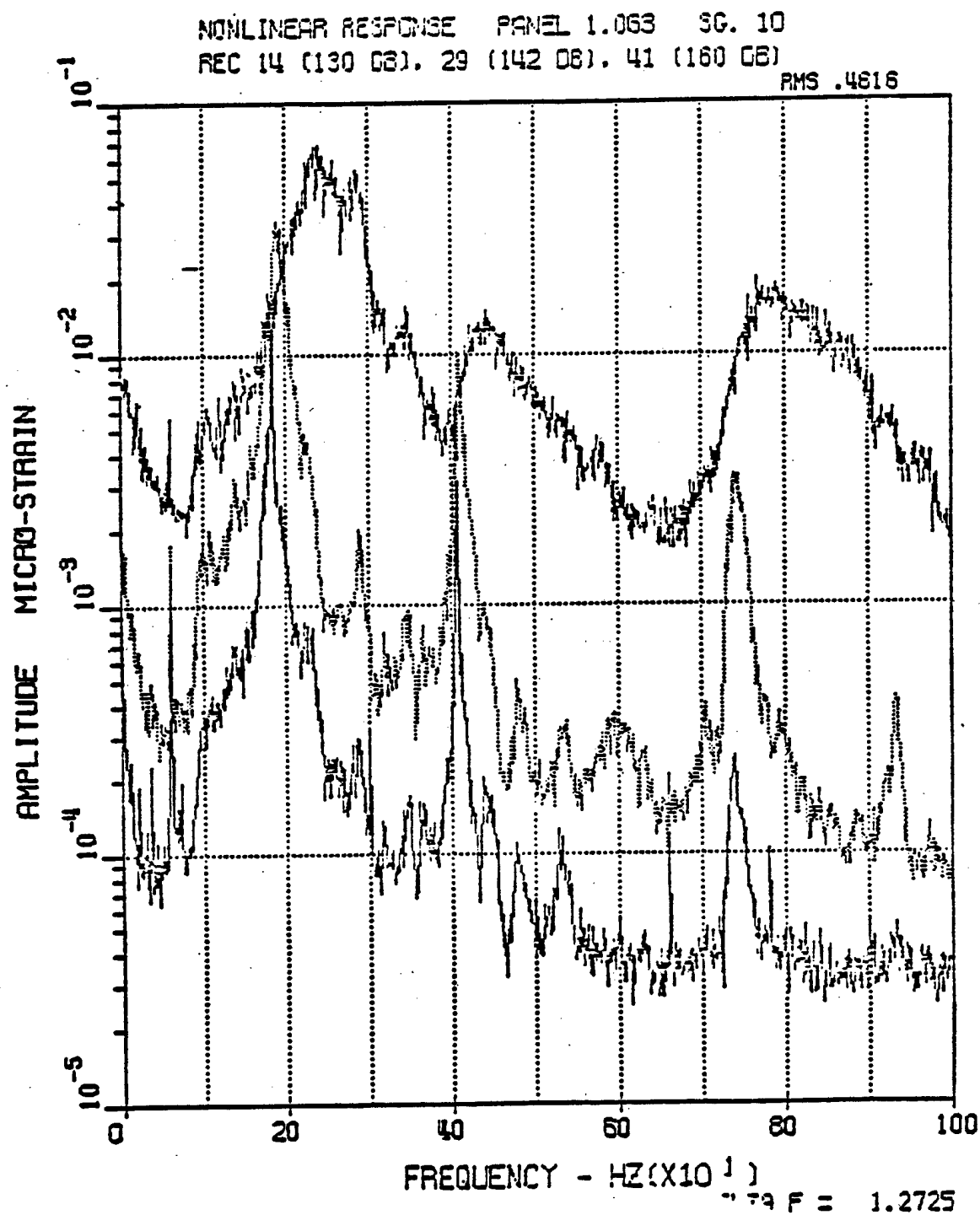


Figure 2. Strain response at three different SPL's.



# NOMENCLATURE

$a, b$	panel length and width
$C$	generalized damping
$D$	flexural rigidity
$E$	Young's modulus
$f_m(x), g_n(y)$	displacement functions, eq. (21)
$F$	airy stress function
$h$	panel thickness
$K$	generalized stiffness
$L$	mathematical operator, eq. (1)
$M$	generalized mass
$p$	pressure
$q$	normal ccoordinate
$r$	length-to-width ratio, $a/b$
$S_p(\omega)$	cross-spectral density of $p(t)$
$t$	time
$w$	lateral deflection
$W$	generalized displacement
$x, y$	coordinates
$\beta$	vector function, eq. (16)
$\zeta$	damping ratio, $c/c_0$
$\phi$	normal mode
$\omega$	linear frequency
$\Omega$	equivalent linear or nonlinear frequency

## Subscripts

EL	equivalent linear
L	linear

# MATHEMATICAL FORMULATION AND SOLUTION PROCEDURE

The governing equations of a rectangular, isotropic plate undergoing large-deflection motions, neglecting the effects of both inplane and rotatory inertia forces, are (refs. 5, 6)

$$\begin{aligned} L(w, F) = & D \nabla^4 w + \rho h w_{,tt} + g w_{,t} \\ & - h (F_{,yy} w_{,xx} + F_{,xx} w_{,yy} - 2 F_{,xy} w_{,xy}) \\ & - p(t) = 0 \end{aligned} \quad (1)$$

$$\nabla^4 F = E (w_{,xy}^2 - w_{,xx} w_{,yy}) \quad (2)$$

where a comma denotes the partial differentiation with respect to the corresponding variable,  $w$  is the lateral deflection,  $F$  is the stress function,  $D$  is the flexural rigidity,  $\rho$  is the mass density,  $h$  is the plate thickness,  $p$  is the pressure,  $E$  is the Young's modulus, and  $g$  is the viscous damping.

The lateral deflection is assumed as

$$w(x, y, t) = h \sum_m \sum_n W_{mn}(t) f_m(x) g_n(y) \quad m, n = 1, 2, 3, \dots \quad (3)$$

where the functions  $f_m(x)$  and  $g_n(y)$  are so chosen that they satisfy the boundary conditions. By solving the compatibility equation, equation (2), the stress function can then be determined as

$$\begin{aligned} F = & \bar{N}_x \frac{y^2}{2} + \bar{N}_y \frac{x^2}{2} + E h^2 \sum_i \sum_j F_{ij} N_i(x) M_j(y) \\ & i, j = 0, 1, 2, \dots \end{aligned} \quad (4)$$

A quasi-exact solution has been obtained by Paul for thermal postbuckling of a clamped, rectangular plate. The expressions for the coefficients  $\bar{N}_x$ ,  $\bar{N}_y$ , and  $F_{ij}$  can be found in reference 7.

Apply the Bubnov-Galerkin method to the equation of motion in deflection, equation (1), as

$$\iint L(w, F) f_r g_s dx dy = 0 \quad r, s = 1, 2, 3, \dots \quad (5)$$

After performing the integration over the total area of the panel, a set of nonlinear, time-differential equations is obtained and can be written in matrix form as

$$[M]\{\ddot{W}\} + [C]\{\dot{W}\} + [K]_L\{W\} + \{\beta(W)\} = \{p(t)\} \quad (6)$$

where the matrices  $[M]$ ,  $[C]$ , and  $[K]_L$  are the generalized mass, damping, and linear stiffness matrices, respectively, and  $\{\beta(W)\}$  is a vector function, cubic in the generalized displacements  $\{W\}$ .

An equivalent linear set of equations to equation (6) may be defined as (refs. 8-13):

$$[M]\{\ddot{W}\} + [C]\{\dot{W}\} + ([K]_L + [K]_{EL})\{W\} = \{p(t)\} \quad (7a)$$

or

$$[M]\{\ddot{W}\} + [C]\{\dot{W}\} + [K]\{W\} = \{p(t)\} \quad (7b)$$

where the elements of the equivalent linear stiffness matrix  $[K]_{EL}$  can be obtained from the expression

$$(K_{EL})_{ij} = E \left[ \frac{\partial \beta_j(W)}{\partial W_i} \right] \quad i, j = 1, 2, 3, \dots \quad (8)$$

where  $E[ ]$  is an expected value operator.

To determine the mean-square generalized displacements  $\bar{w}_j^2$  in equation (7), an iterative solution procedure is introduced. The undamped linear equation of equation (7a) is solved first. This requires the determination of the eigenvalues and eigenvectors of the undamped linear equation

$$\omega_j^2 [M] \{\phi\}_j = [K]_L \{\phi\}_j \quad (9)$$

where  $\omega_j$  is the frequency of vibration and  $\{\phi\}_j$  is the corresponding normal mode shape based on linear theory.

Apply a coordinate transformation, from the generalized displacements to the normal coordinates, by

$$\begin{matrix} \{W\} &= [\phi] \{q\} & n \leq m \\ m \times 1 & m \times n \quad n \times 1 \end{matrix} \quad (10)$$

in which each column of  $[\phi]$  is a modal column of the linear system, and  $\{q\}$  represents the normal coordinates. Substituting equation (10) into the damped linear equation of equation (7a) and premultiplying by the transpose of  $[\phi]$ , it becomes

$$[\bar{M}] \ddot{\{q\}} + [\bar{C}] \dot{\{q\}} + [\bar{K}]_L \{q\} = \{P(t)\} \quad (11)$$

where  $[\bar{M}] = [\phi]^T [M] [\phi]$

$$[\bar{K}]_L = [\phi]^T [K]_L [\phi] = [\omega^2] [\bar{M}]$$

$$[\bar{C}] = [\phi]^T [C] [\phi] = 2[\zeta\omega] [\bar{M}]$$

$$\{P\} = [\phi]^T \{p\} \quad (12)$$

The  $j$ th row of equation (11) is

$$\ddot{q}_j + 2\zeta_j \omega_j \dot{q}_j + \omega_j^2 q_j = \frac{P_j}{M_j} \quad (13)$$

The mean-square normal coordinate is simply

$$\overline{q_j^2} = \frac{\pi S_p(\omega_j)}{4 M_j^2 \zeta_j \omega_j^3} = \frac{\pi \{\phi\}_j^T [S_p(\omega_j)] \{\phi\}_j}{4 M_j^2 \zeta_j \omega_j^3} \quad (14)$$

where  $[S_p]$  is the cross-spectral density matrix of the excitation  $\{p(t)\}$ . The covariance matrix of the linear, generalized displacements is

$$[\overline{W_i W_j}]_L = \sum_k \{\phi\}_k \frac{\pi \{\phi\}_k^T [S_p(\omega_k)] \{\phi\}_k}{4 M_k^2 \zeta_k \omega_k^3} \{\phi\}_k^T \quad (15)$$

The diagonal terms  $[\overline{W_i W_j}]_L$  are the mean-square, linear, generalized displacement  $\overline{W_j^2}$ . This initial estimate of  $\overline{W_j^2}$  can now be used to compute the equivalent linear stiffness matrix  $[K]_{EL}^j$  through equation (8). Then equation (7) is again transformed to the normal coordinates and has the form as

$$[M] \{\ddot{q}\} + [C] \{\dot{q}\} + [K] \{q\} = \{P(t)\} \quad (16)$$

$$\text{where } [K] = [\phi]^T ([K]_L + [K]_{EL}) [\phi] = [\Omega^2] [M] \quad (17)$$

The  $j$ th row of equation (17) is

$$\ddot{q} + 2\zeta_j \omega_j \dot{q}_j + \Omega^2 q = \frac{P_j}{M_j} \quad (18)$$

and the displacement covariance matrix is given by

$$[\overline{W_i W_j}] = \sum_k \{\phi\}_k \frac{\pi \{\phi\}_k^T [S_p(\Omega_k)] \{\phi\}_k}{4 M_k^2 \zeta_k \Omega_k^2 \omega_k} \{\phi\}_k^T \quad (19)$$

Convergence is considered achieved whenever the difference of the RMS generalized displacements satisfies the requirement

$$\left| \frac{(\text{RMS } W_j)_{\text{iter}} - (\text{RMS } W_j)_{\text{iter}-1}}{(\text{RMS } W_j)_{\text{iter}}} \right| < 10^{-3}, \text{ for all } j \quad (20)$$

Once the RMS displacements are determined, the RMS deflection of the panel and the maximum RMS strain can be determined from equation (3) and the strain-displacement relations, respectively.

DEVELOPMENT OF GENERALIZED MATRICES  
AND COMPUTER PROGRAMS

The deflection of the panel is represented by

$$w(x,y,t) = h \sum_m \sum_n W_{mn}(t) \left\{ \left[ \cos \frac{(m-1)\pi x}{a} - \cos \frac{(m+1)\pi x}{a} \right] \cdot \left[ \cos \frac{(n-1)\pi y}{b} - \cos \frac{(n+1)\pi y}{b} \right] \right\} \quad (21)$$

The expression  $w$  satisfies the boundary condition for clamped edges:

$$\begin{aligned} w = w_{,x} = 0 & \quad \text{on } x = 0 \text{ and } a \\ w = w_{,y} = 0 & \quad \text{on } y = 0 \text{ and } b \end{aligned} \quad (22)$$

The stress function can be expressed in terms of the generalized displacement  $W_{mn}$  as

$$F = \bar{N}_x \frac{y^2}{2} + \bar{N}_y \frac{x^2}{2} + Eh^2 \sum_i \sum_j F_{ij} \cos \frac{i\pi x}{a} \cos \frac{j\pi y}{b} \quad (23)$$

where the coefficient  $F_{ij}$  is given by the expression

$$F_{ij} = \frac{1}{\left(\frac{i^2}{r} + j^2 r\right)^2} \sum_m \sum_n \sum_k \sum_l B_{ijmnkl} W_{mn} W_{kl} \quad (24)$$

in which  $B_{ijmnkl}$  are integers and  $r = a/b$ . The coefficients  $\bar{N}_x$  and  $\bar{N}_y$ , and the integers  $B_{ijmnkl}$  are given explicitly in reference 7. The particular generalized displacements that are chosen to be nonzero in the convergence studies are shown in table 1.

Table 1. Generalized displacements for convergence studies.

<u>Generalized Displacements</u>	<u>Number of terms</u>				
	<u>1</u>	<u>4</u>	<u>6</u>	<u>10</u>	<u>15</u>
$W_{11}$	X	X	X	X	X
$W_{13}$		X	X	X	X
$W_{31}$		X	X	X	X
$W_{33}$		X	X	X	X
$W_{15}$			X	X	X
$W_{51}$			X	X	X
$W_{35}$				X	X
$W_{53}$				X	X
$W_{17}$				X	X
$W_{71}$				X	X
$W_{55}$					X
$W_{37}$					X
$W_{73}$					X
$W_{19}$					X
$W_{91}$					X



Utilizing the expressions for  $w$  and  $F$ , equations (21) and (23), respectively, and performing the integration of equation (5), the integral associated with the inertial force term in equation (1) has been derived as

$$\int_0^b \int_0^a \rho h w_{,tt} f_r g_s dx dy = \frac{\rho h^2 ab}{4} \left( \ddot{w}_{r-2,s-2} - 2 \ddot{w}_{r-2,s} - 2 \ddot{w}_{r,s-2} + 4 \ddot{w}_{r,s} - 2 \ddot{w}_{r,s+2} - 2 \ddot{w}_{r+2,s} + \ddot{w}_{r-2,s+2} + \ddot{w}_{r+2,s-2} + \ddot{w}_{r+2,s+2} \right) \quad (25)$$

The generalized mass matrix  $[M]$  in equation (6) using 15 terms in the deflection function is given by:

$$[M] = \frac{\rho h^2 ab}{4} \begin{matrix} & \begin{matrix} W_{11} & W_{13} & W_{31} & W_{33} & W_{15} & W_{51} & W_{35} & W_{53} & W_{17} & W_{71} & W_{55} & W_{37} & W_{73} & W_{19} & W_{91} \end{matrix} \\ \begin{bmatrix} 4 \\ -2 & 4 \\ -2 & 1 & 4 \\ 1 & -2 & -2 & 4 \\ 0 & -2 & 0 & 1 & 4 \\ 0 & 0 & -2 & 1 & 0 & 4 \\ 0 & 1 & 0 & -2 & -2 & 0 & 4 \\ 0 & 0 & 1 & -2 & 0 & -2 & 1 & 4 \\ 0 & 0 & 0 & 0 & -2 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 0 & 0 & -2 & -2 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 0 & -2 & 0 & -2 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 & 0 & -2 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 1 & 0 & 4 \end{bmatrix} \end{matrix} \quad (26)$$

A subroutine program MASS, which generates the mass matrix, has been coded and verified. A listing of the MASS subroutine is given in the Appendix.

Similarly, the integrals associated with the linear stiffness terms in equation (1) yield

$$\int_0^b \int_0^a D \frac{\partial^4 w}{\partial x^4} f_r g_s dx dy = \frac{Dh\pi^4 ab}{4a^4}$$

$$\cdot \{ [(r-1)^4 + (r+1)^4] [(C_1 + 1)W_{r,s} - W_{r,s-2} - W_{r,s+2}]$$

$$+ (r-1)^4 [W_{r-2,s-2} + W_{r-2,s+2} - (C_1 + 1)W_{r-2,s}]$$

$$+ (r+1)^4 [W_{r+2,s+2} + W_{r+2,s-2} - (C_1 + 1)W_{r+2,s}] \} \quad (27)$$

$$\int_0^b \int_0^a D \frac{\partial^4 w}{\partial y^4} f_r g_s dx dy = \frac{Dh\pi^4 ab}{4b^4}$$

$$\cdot \{ [(s-1)^4 + (s+1)^4] [(C_2 + 1)W_{r,s} - W_{r-2,s} - W_{r+2,s}]$$

$$+ (s-1)^4 [W_{r-2,s-2} + W_{r+2,s-2} - (C_2 + 1)W_{r,s-2}]$$

$$+ (s+1)^4 [W_{r+2,s+2} + W_{r-2,s+2} - (C_2 + 1)W_{r,s+2}] \} \quad (28)$$

$$\int_0^b \int_0^a 2D \frac{\partial^4 w}{\partial x^2 \partial y^2} f_r g_s dx dy = \frac{Dh\pi^4 ab}{a^2 b^2}$$

$$\cdot \{ (r-1)^2 (s+1)^2 [W_{r,s} - W_{r,s+2} - W_{r+2,s} + W_{r+2,s+2}]$$

$$+ (r-1)^2 (s+1)^2 [W_{r,s} - W_{r,s+2} - W_{r-2,s} + W_{r-2,s+2}]$$

$$+ (r+1)^2 (s-1)^2 [W_{r,s} - W_{r,s-2} - W_{r+2,s} + W_{r+2,s-2}]$$

$$+ (r+1)^2 (s-1)^2 [W_{r,s} - W_{r,s-2} - W_{r-2,s} + W_{r-2,s-2}] \} \quad (29)$$

$$[K]_L = \frac{Dh\pi^4 ab}{4} \left( \frac{1}{a^4} [K]_1 + \frac{1}{b^4} [K]_2 + \frac{4}{a^2 b^2} [K]_3 \right) \quad (30)$$
$$[K]_1 = \begin{bmatrix} w_{11} & w_{13} & w_{31} & w_{33} & w_{15} \\ 2^4 (C_1 + 1) & & & & \\ -2^4 & 2^4 (C_1 + 1) & & & \\ -2^4 (C_1 + 1) & 2^4 & (2^4 + 4^4) (C_1 + 1) & & \\ 2^4 & -2^4 (C_1 + 1) & -(2^4 + 4^4) & (2^4 + 4^4) (C_1 + 1) & \\ 0 & -2^4 & 0 & 2^4 & 2^4 (C_1 + 1) \\ 0 & 0 & -4^4 (C_1 + 1) & 4^4 & 0 \\ 0 & 2^4 & 0 & -(2^4 + 4^4) & -2^4 (C_1 + 1) \\ 0 & 0 & 4^4 & -4^4 (C_1 + 1) & 0 \\ 0 & 0 & 0 & 0 & -2^4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$[K]_2 = \begin{bmatrix} W_{11} & W_{13} & W_{31} & W_{33} & W_{15} \\ 2^4(C_2+1) & & & & \\ -2^4(C_2+1) & (2^4+4^4)(C_2+1) & & & \\ -2^4 & 2^4 & 2^4(C_2+1) & & \\ 2^4 & -(2^4+4^4) & -2^4(C_1+1) & (2^4+4^4)(C_2+1) & \\ 0 & -4^4(C_2+1) & 0 & 4^4 & (4^4+6^4)(C_2+1) \\ 0 & 0 & -2^4 & 2^4 & 0 \\ 0 & 4^4 & 0 & -4^4(C_2+1) & -(4^4+6^4) \\ 0 & 0 & 2^4 & -(2^4+4^4) & 0 \\ 0 & 0 & 0 & 0 & -6^4(C_2+1) \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} W_{51} & W_{35} & W_{53} & W_{17} & W_{71} \\ 2^4(C_2+1) & & & & \\ 0 & (4^4+6^4)(C_2+1) & & \text{symmetric} & \\ -2^4(C_2+1) & 4^4 & (2^4+4^4)(C_2+1) & & \\ 0 & 6^4 & 0 & (6^4+8^4)(C_1+1) & \\ -2^4 & 0 & 2^4 & 0 & 2^4(C_1+1) \end{bmatrix}$$

(32)

$$[K]_3 = \begin{bmatrix} W_{11} & W_{13} & W_{31} & W_{33} & W_{15} \\ (2 \cdot 2)^2 & & & & \\ -(2 \cdot 2)^2 & (2 \cdot 4)^2 + (2 \cdot 2)^2 & & \text{symmetric} & \\ -(2 \cdot 2)^2 & (2 \cdot 2)^2 & (4 \cdot 2)^2 + (2 \cdot 2)^2 & (4 \cdot 4)^2 & \\ (2 \cdot 2)^2 & -(2 \cdot 4)^2 - (2 \cdot 2)^2 & -(4 \cdot 2)^2 - (2 \cdot 2)^2 & +2(4 \cdot 2)^2 + (2 \cdot 2)^2 & \\ 0 & -(2 \cdot 4)^2 & 0 & (2 \cdot 4)^2 & (2 \cdot 6)^2 + (2 \cdot 4)^2 \\ 0 & 0 & -(4 \cdot 2)^2 & (4 \cdot 2)^2 & 0 \\ 0 & (2 \cdot 4)^2 & 0 & -(4 \cdot 4)^2 - (2 \cdot 4)^2 & -(2 \cdot 6)^2 - (2 \cdot 4)^2 \\ 0 & 0 & (4 \cdot 2)^2 & -(4 \cdot 4)^2 - (4 \cdot 2)^2 & 0 \\ 0 & 0 & 0 & 0 & -(2 \cdot 6)^2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} W_{51} & W_{35} & W_{53} & W_{17} & W_{71} \\ (6 \cdot 2)^2 + (4 \cdot 2)^2 & & & & \\ 0 & (4 \cdot 6)^2 + (2 \cdot 6)^2 + (4 \cdot 4)^2 + (2 \cdot 4)^2 & & & \\ -(6 \cdot 2)^2 - (4 \cdot 2)^2 & (4 \cdot 4)^2 & (6 \cdot 4)^2 + (4 \cdot 4)^2 + (6 \cdot 2)^2 + (4 \cdot 2)^2 & & \\ 0 & (2 \cdot 6)^2 & 0 & (2 \cdot 8)^2 + (2 \cdot 6)^2 & \\ -(6 \cdot 2)^2 & 0 & (6 \cdot 2)^2 & 0 & (8 \cdot 2)^2 + (6 \cdot 2)^2 \end{bmatrix} \quad (33)$$

The nonzero elements of the linear stiffness matrix,  $k_L(i,j)$  for  $i,j \geq 11$ , are

$$\begin{aligned} k_1(11,4) &= k_1(12,11) = 4^4 \\ k_1(12,5) &= k_1(14,12) = 2^4 \\ k_1(13,6) &= k_1(13,11) = 6^4 \\ k_1(11,7) &= -4^4(C_1 + 1) \\ k_1(12,7) &= -(2^4 + 4^4) \\ k_1(11,8) &= -(4^4 + 6^4) \end{aligned} \quad \begin{matrix} (34) \\ (\text{cont'd}) \end{matrix}$$

$$\begin{aligned}
k_1(13,8) &= -6^4 (C_1 + 1) \\
k_1(12,9) &= -2^4 (C_1 + 1) \\
k_1(14,9) &= -2^4 \\
k_1(13,10) &= -(6^4 + 8^4) \\
k_1(15,10) &= -8^4 (C_1 + 1) \\
k_1(11,11) &= (4^4 + 6^4)(C_1 + 1) \\
k_1(12,12) &= (2^4 + 4^4)(C_1 + 1) \\
k_1(13,13) &= (6^4 + 8^4)(C_1 + 1) \\
k_1(15,13) &= 8^4 \\
k_1(14,14) &= 2^4 (C_1 + 1) \\
k_1(15,15) &= (8^4 + 10^4)(C_1 + 1) \\
k_2(11,4) &= k_2(13,11) = 4^4 \\
k_2(12,5) &= k_2(12,11) = 6^4 \\
k_2(13,6) &= k_2(15,13) = 2^4 \\
k_2(11,7) &= -(4^4 + 6^4) \\
k_2(12,7) &= -6^4 (C_2 + 1) \\
k_2(11,8) &= -4^4 (C_2 + 1) \\
k_2(13,8) &= -(2^4 + 4^4) \\
k_2(12,9) &= -(6^4 + 8^4) \\
k_2(14,9) &= -8^4 (C_2 + 1) \\
k_2(13,10) &= -2^4 (C_2 + 1) \\
k_2(15,10) &= -2^4 \\
k_2(11,11) &= (4 + 6)(C_2 + 1) \\
k_2(12,12) &= (6^4 + 8^4)(C_2 + 1) \\
k_2(14,12) &= 8^4 \\
k_2(13,13) &= (2^4 + 4^4)(C_2 + 1) \\
k_2(14,14) &= (8^4 + 10^4)(C_2 + 1) \\
k_2(15,15) &= 2^4 (C_2 + 1) \\
k_3(11,4) &= (4 \cdot 4)^2 \\
k_3(12,5) &= k_3(13,6) = (2 \cdot 6)^2 \\
k_3(11,7) &= k_3(11,8) = -(4 \cdot 4)^2 - (4 \cdot 6)^2 \\
k_3(12,7) &= k_3(13,8) = -(2 \cdot 6)^2 - (4 \cdot 6)^2 \\
k_3(12,9) &= k_3(13,10) = -(2 \cdot 6)^2 - (2 \cdot 8)^2 \\
k_3(14,9) &= k_3(15,10) = -(2 \cdot 8)^2 \\
k_3(11,11) &= (6 \cdot 6)^2 + 2(6 \cdot 4)^2 + (4 \cdot 4)^2 \\
k_3(12,11) &= k_3(13,11) = (4 \cdot 6)^2
\end{aligned}$$

(34)

(cont'd)

$$\begin{aligned}
 k_3(12,12) &= k_3(13,13) = (4 \cdot 8)^2 + (2 \cdot 8)^2 + (4 \cdot 6)^2 + (2 \cdot 6)^2 & (34) \\
 k_3(14,14) &= k_3(15,15) = (2 \cdot 8)^2 + (2 \cdot 10)^2 & (\text{concl'd})
 \end{aligned}$$

where

$$\begin{aligned}
 C_1 &= \begin{matrix} 2 \\ 1 \end{matrix} \quad \begin{cases} \text{for } s = 1 \\ \text{for } s \neq 1 \end{cases} \\
 C_2 &= \begin{matrix} 2 \\ 1 \end{matrix} \quad \begin{cases} \text{for } r = 1 \\ \text{for } r \neq 1 \end{cases} & (35)
 \end{aligned}$$

A subroutine program LSTF which generates the linear stiffness matrix has been coded and verified. A listing of the LSTF subroutine is presented in the Appendix.

Derivation of the generalized equivalent linear stiffness matrix  $[K]_{EL}$  in equation (7a) has been initiated. It is in good progress. Continuing research effort will be devoted to the following tasks:

- (1) Completion of the derivation of equivalent linear stiffness;
- (2) Application of eigen solution and coordinate transformation;
- (3) Determination of mean-square linear generalized displacement;
- (4) Implementation of the iterative process;
- (5) Derivation of strains computation;
- (6) Coding, debugging, and verifying the complete computer program;
- (7) Convergence studies; and
- (8) Generation of design charts.<sup>7</sup>

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## APPENDIX

### LISTINGS OF THE MASS AND LSTF SUBROUTINES

```

00001 SUBROUTINE MASS(AL,BL,H,RHO,SM,NTERM)
00002 DIMENSION SM(NTERM,NTERM)
00003 C
00004 C - THIS SUBROUTINE GENERATES THE SYSTEM MASS MATRIX OF THE PANEL USING
00005 C (NTERM) TERMS IN THE DEFLECTION FUNCTION
00006 C AL=PANEL LENGTH.
00007 C BL=PANEL WIDTH
00008 C H=PANEL THICKNESS
00009 C RHO=MASS DENSITY
00010 C SM(NTERM,NTERM)=SYSTEM OR GENERALIZED MASS MATRIX
00011 C NTERM=1, 4, 6, 10, Or 15
00012 C
00013 COEF=0.25*RHO*H*H*AL*BL
00014 C INITIALIZED THE MASS MATRIX
00015 DO 10 I=1,NTERM
00016 DO 10 J=1,NTERM
00017 SM(I,J)=0.0
00018 10 CONTINUE
00019 SM(1,1)=4.0*COEF
00020 IF (NTERM.EQ. 1) GO TO 20
00021 C
00022 SM(1,2)=-2.0*COEF
00023 SM(1,3)=-2.0*COEF
00024 SM(1,4)=COEF
00025 SM(2,2)=4.0*COEF
00026 SM(2,3)=COEF
00027 SM(2,4)=-2.0*COEF
00028 SM(3,3)=4.0*COEF
00029 SM(3,4)=-2.0*COEF
00030 SM(4,4)=4.0*COEF
00031 IF (NTERM.EQ. 4) GO TO 20
00032 C
00033 SM(2,5)=-2.0*COEF
00034 SM(3,6)=-2.0*COEF
00035 SM(4,5)=COEF
00036 SM(4,6)=COEF
00037 SM(5,5)=4.0*COEF
00038 SM(6,6)=4.0*COEF
00039 IF (NTERM.EQ. 6) GO TO 20
00040 C
00041 SM(2,7)=COEF
00042 SM(3,8)=COEF
00043 SM(4,7)=-2.0*COEF
00044 SM(4,8)=-2.0*COEF
00045 SM(5,7)=-2.0*COEF
00046 SM(5,9)=-2.0*COEF
00047 SM(6,8)=-2.0*COEF
00048 SM(6,10)=-2.0*COEF
00049 SM(7,7)=4.0*COEF
00050 SM(7,8)=COEF
00051 SM(7,9)=COEF
00052 SM(8,8)=4.0*COEF
00053 SM(8,10)=COEF
00054 SM(9,9)=4.0*COEF
00055 SM(10,10)=4.0*COEF
00056 IF (NTERM.EQ. 10) GO TO 20
00057

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1 00057 C
2 00058 SM(4,11)=COEF
3 00059 SM(5,12)=COEF
4 00060 SM(6,13)=COEF
5 00061 SM(7,11)=-2.0*COEF
6 00062 SM(7,12)=-2.0*COEF
7 00063 SM(8,11)=-2.0*COEF
8 00064 SM(8,13)=-2.0*COEF
9 00065 SM(9,12)=-2.0*COEF
10 00066 SM(9,14)=-2.0*COEF
11 00067 SM(10,13)=-2.0*COEF
12 00068 SM(10,15)=-2.0*COEF
13 00069 SM(11,11)=4.0*COEF
14 00070 SM(11,12)=COEF
15 00071 SM(11,13)=COEF
16 00072 SM(12,12)=4.0*COEF
17 00073 SM(12,14)=COEF
18 00074 SM(13,13)=4.0*COEF
19 00075 SM(13,15)=COEF
20 00076 SM(14,14)=4.0*COEF
21 00077 SM(15,15)=4.0*COEF
22 00078 20 CONTINUE
23 00079 DO 30 J=1,NTERM
24 00080 DO 30 I=J,NTERM
25 00081 SM(I,J)=SM(J,I)
26 00082 30 CONTINUE
27 00083 RETURN
28 00084 END,
29
30
31 SUBPROGRAMS CALLED
32
33
34
35 SCALARS AND ARRAYS [ ** NO EXPLICIT DEFINITION - *% NOT REFERENCED ]
36
37 *NTERM 1 *COEF 2 *H 3 *HL 4 SM 5
38 *J 6 .S0003 7 .S0002 10 .S0001 11 .S0000 12
39 *RHO 13 *AL 14 .I0002 15 .I0001 16 *I 17
40 .I0000 20
41
42 TEMPORARIES
43
44 .A0016 21
45
46 MASS [ NO ERRORS DETECTED ]
47
48
49
50
51
52
53
54
55
56
57

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1 00001 SUBROUTINE LSTF(AL,BL,H,D,PI,SK,NTERM)
2 00002 DIMENSION SK(NTERM,NTERM)
3 00003 C
4 00004 C THIS SUBROUTINE GENERATES THE SYSTEM LINEAR STIFFNESS MATRIX
5 00005 C USING (NTERM) TERMS IN THE DEFLECTION FUNCTION FOR THE PANEL
6 00006 C AL=PANEL LENGTH
7 00007 C BL=PANEL WIDTH
8 00008 C H=PANEL THICKNESS
9 00009 C D=E*H**3/((12.0*(1.0-V**2)))=BENDING RIGIDITY
10 00010 C V=POISSON'S RATIO
11 00011 C SK(NTERM,NTERM)=SYSTEM OR GENERALIZED STIFFNESS MATRIX
12 00012 C NTERM=1, 4, 6, 10, OR 15
13 00013 C
14 00014 PI4=PI*PI*PI*PI
15 00015 COEF=4.0*D*H*PI4*AL*BL
16 00016 A4=1.0/(AL*AL*AL*AL)
17 00017 B4=1.0/(BL*BL*BL*BL)
18 00018 AB2=4.0/(AL*AL*BL*BL)
19 00019 C C1=C1 FOR S<>1,C1=CC1 FOR S=1
20 00020 C C2=C2 FOR R<>1,C2=CC2 FOR R=1
21 00021 C1=1.0
22 00022 C2=1.0
23 00023 CC1=2.0
24 00024 CC2=2.0
25 00025 C INITIALIZED THE STIFFNESS MATRIX
26 00026 DO 10 I=1,NTERM
27 00027 DO 10 J=1,NTERM
28 00028 SK(I,J)=0.0
29 00029 10 CONTINUE
30 00030 SK(1,1)=((CC1+1.0)*A4+(CC2+1.0)*B4+AB2)*COEF
31 00031 IF(NTERM.EQ. 1) GO TO 20
32 00032 C
33 00033 SK(1,2)=-((CC2+1.0)*B4+A4+AB2)*COEF
34 00034 SK(1,3)=-((CC1+1.0)*A4+B4+AB2)*COEF
35 00035 SK(1,4)=(A4+B4+AB2)*COEF
36 00036 SK(2,2)=(17.0*(CC2+1.0)*B4+(C1+1.0)*A4+5.0*AB2)*COEF
37 00037 SK(2,3)=(A4+B4+AB2)*COEF
38 00038 SK(2,4)=-((C1+1.0)*A4+17.0*B4+5.0*AB2)*COEF
39 00039 SK(3,3)=(17.0*(CC1+1.0)*A4+(C2+1.0)*B4+5.0*AB2)*COEF
40 00040 SK(3,4)=-((C2+1.0)*B4+17.0*A4+5.0*AB2)*COEF
41 00041 SK(4,4)=(17.0*(C1+1.0)*A4+17.0*(C2+1.0)*B4+25.0*AB2)*COEF
42 00042 IF(NTERM.EQ. 4) GO TO 20
43 00043 C
44 00044 SK(2,5)=-((16.0*(CC2+1.0)*B4+A4+4.0*AB2)*COEF
45 00045 SK(3,6)=-((16.0*(CC1+1.0)*A4+B4+4.0*AB2)*COEF
46 00046 SK(4,5)=(A4+16.0*B4+4.0*AB2)*COEF
47 00047 SK(4,6)=(16.0*A4+B4+4.0*AB2)*COEF
48 00048 SK(5,5)=((C1+1.0)*A4+97.0*(CC2+1.0)*B4+13.0*AB2)*COEF
49 00049 SK(6,6)=(97.0*(CC1+1.0)*A4+(C2+1.0)*B4+13.0*AB2)*COEF
50 00050 IF(NTERM.EQ. 6) GO TO 20
51 00051 C
52 00052 SK(2,7)=(A4+16.0*B4+4.0*AB2)*COEF
53 00053 SK(3,8)=(16.0*A4+B4+4.0*AB2)*COEF
54 00054 SK(4,7)=-((17.0*A4+16.0*(C2+1.0)*B4+20.0*AB2)*COEF
55 00055 SK(4,8)=-((16.0*(C1+1.0)*A4+17.0*B4+20.0*AB2)*COEF
56 00056 SK(5,7)=-((C1+1.0)*A4+97.0*B4+13.0*AB2)*COEF
57

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00057      SK(5,9)=- (A4+81.0*(CC2+1.0)*B4+9.0*AB2)*COEF
00058      SK(6,8)=- (97.0*A4+(C2+1.0)*B4+13.0*AB2)*COEF
00059      SK(6,10)=- (81.0*(CC1+1.0)*A4+B4+9.0*AB2)*COEF
00060      SK(7,7)= (17.0*(C1+1.0)*A4+97.0*(C2+1.0)*B4+65.0*AB2)*COEF
00061      SK(7,8)= 16.0*(A4+B4+AB2)*COEF
00062      SK(7,9)= (A4+81.0*B4+9.0*AB2)*COEF
00063      SK(8,8)= (97.0*(C1+1.0)*A4+17.0*(C2+1.0)*B4+65.0*AB2)*COEF
00064      SK(8,10)= (81.0*A4+B4+9.0*AB2)*COEF
00065      SK(9,9)= ((C1+1.0)*A4+337.0*(CC2+1.0)*B4+25.0*AB2)*COEF
00066      SK(10,10)= (337.0*(CC1+1.0)*A4+(C2+1.0)*B4+25.0*AB2)*COEF
00067      IF(NTERM.EQ.10) GO TO 20
00068      C
00069      SK(1,11)= 16.0*(A4+B4+AB2)*COEF
00070      SK(5,12)= (A4+81.0*B4+9.0*AB2)*COEF
00071      SK(6,13)= (B4+81.0*A4+9.0*AB2)*COEF
00072      SK(7,11)= - (16.0*(C1+1.0)*A4+97.0*B4+52.0*AB2)*COEF
00073      SK(7,12)= - (17.0*A4+81.0*(C2+1.0)*B4+45.0*AB2)*COEF
00074      SK(8,11)= - (97.0*A4+16.0*(C2+1.0)*B4+52.0*AB2)*COEF
00075      SK(8,13)= - (81.0*(C1+1.0)*A4+17.0*B4+45.0*AB2)*COEF
00076      SK(9,12)= - ((C1+1.0)*A4+337.0*B4+25.0*AB2)*COEF
00077      SK(9,14)= - (A4+256.0*(CC2+1.0)*B4+16.0*AB2)*COEF
00078      SK(10,13)= - (337.0*A4+(C2+1.0)*B4+25.0*AB2)*COEF
00079      SK(10,15)= - (256.0*(CC1+1.0)*A4+B4+16.0*AB2)*COEF
00080      SK(11,11)= (97.0*(C1+1.0)*A4+97.0*(C2+1.0)*B4+169.0*AB2)*COEF
00081      SK(11,12)= (16.0*A4+81.0*B4+36.0*AB2)*COEF
00082      SK(11,13)= (81.0*A4+16.0*B4+36.0*AB2)*COEF
00083      SK(12,12)= (17.0*(C1+1.0)*A4+337.0*(C2+1.0)*B4+125.0*AB2)*COEF
00084      SK(12,14)= (A4+256.0*B4+16.0*AB2)*COEF
00085      SK(13,13)= (337.0*(C1+1.0)*A4+17.0*(C2+1.0)*B4+125.0*AB2)*COEF
00086      SK(13,15)= (B4+256.0*A4+16.0*AB2)*COEF
00087      SK(14,14)= ((C1+1.0)*A4+881.0*(CC2+1.0)*B4+41.0*AB2)*COEF
00088      SK(15,15)= ((C2+1.0)*B4+881.0*(CC1+1.0)*A4+41.0*AB2)*COEF
00089      20 CONTINUE
00090      DO 30 J=1,NTERM
00091      DO 30 I=J,NTERM
00092      SK(I,J)=SK(J,I)
00093      30 CONTINUE
00094      RETURN
00095      END

```

## SUBPROGRAMS CALLED

## SCALARS AND ARRAYS [ "\*" NO EXPLICIT DEFINITION = "?" NOT REFERENCED ]

NAME	1	NAME	2	NAME	3	NAME	4	NAME	5
*A4	1	*NTERM	2	*COEF	3	*PI	4	*H	5
*CC1	6	*BL	7	*J	10	*D	11	*AB2	12
.S0003	13	.S0002	14	.S0001	15	.S0000	16	*B4	17
*CC2	20	*C2	21	*PI4	22	*AL	23	SK	24
.I0002	25	.I0001	26	*I	27	.I0000	30	*C1	31